

# MICROWAVE ATTENUATION IN RAIN; A DUAL WAVELENGTH RADAR STUDY

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## ABSTRACT

Microwave attenuation in rain at C-band, observed in radar data from Alberta, Canada, has been assessed by means of two different estimators: one employing  $K_{DP}$  measured simultaneously by an S-band radar and the other a comparison of synchronised S- and C-band reflectivity measurements. While good agreement between the two has been found in some cases, in others the second method gave a larger estimate than the first by up to a factor of two. It is possible that the  $K_{DP}$  method is underestimating the attenuation due to the assumption of a Marshall-Palmer drop size distribution which does not accurately enough represent the true drop size distribution present.

**Keywords :** Attenuation. Microwave. Rain. Radar

## 1 INTRODUCTION

Knowledge of microwave attenuation in rain is important not only for predicting communication link performance, but also for interpreting radar images. In particular, the role of attenuation at C-band is important for interpreting data from many operational radars. This paper aims to investigate C-band attenuation by analysis of data from two co-located, synchronised radars in Alberta, Canada. One of these radars is a conventional, linearly polarised C-band (5.635 GHz) radar while the other is an S-band (2.88 GHz) radar employing circular polarization diversity. If the C-band attenuation can be successfully estimated from the polarisation information at S-band, in particular  $K_{DP}$ , then it should be possible to extend the approach to estimate attenuation at other frequencies.

## 2 ESTIMATION OF C-BAND ATTENUATION

In a region filled with rain, with a drop size distribution  $N(D)$ , horizontally ( $H$ ) and vertically ( $V$ ) polarized waves have propagation constants given by

$$\kappa_{H,V} = k_0 + \frac{2\pi}{k_0} \int f_{H,V}(D) N(D) dD \quad (1)$$

where  $k_0 = \frac{2\pi}{\lambda}$ , where  $\lambda$  is the free space wavelength of the radar, and  $f_{H,V}$  are the appropriate forward scattering amplitudes. For ground-based radars and raindrops of oblate spheroidal shape, as is thought to be the case for all but the smallest drops,  $\kappa_H$  and  $\kappa_V$  will be different. This leads to both a differential attenuation and a differential propagation phase shift. It is the latter which is of interest here.

The two-way differential propagation phase shift  $\phi_{DP}$  through a uniformly rain-filled region of length  $L$  is given by

$$\phi_{DP} = 2 \times \frac{180}{\pi} \text{Re}(\kappa_H - \kappa_V) \times L \quad (2)$$

Unlike measurements of backscatter quantities, such as reflectivity, which relate to a single bin,  $\phi_{DP}$  is accumulated over increasing numbers of bins. We may, however, derive the quantity  $K_{DP}$  which relates to a single bin by using

$$K_{DP}(j) = \frac{\phi_{DP}(j) - \phi_{DP}(j-1)}{2\Delta} \quad (3)$$

where  $\phi_{DP}(j)$  refers to the differential propagation phase shift along a path from the radar to the  $j$ th bin and  $\Delta$  is the bin length.  $K_{DP}$  is the specific one way differential propagation phase shift and it is from this quantity that the C-band attenuation has been estimated using an empirical relationship derived by Tan [1]. This relationship is

$$a_H = 0.248 K_{DP}^{0.856} \quad (\text{dB/km}) \quad (4)$$

where  $a_H$  is the specific C-band horizontal attenuation and  $K_{DP}$  is at S-band. A Marshall-Palmer drop size distribution (DSD) [2], drop

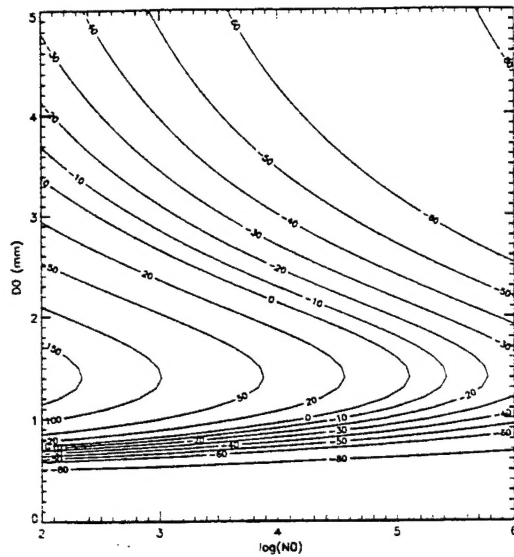


Figure 1: Percentage error in attenuation estimate

shapes deduced from measurements by Goddard and Cherry [3] and Pruppacher and Pitter [4] and a temperature of  $0^{\circ}\text{C}$  have been assumed in the derivation of this equation. The most important of these assumptions is that relating to the drop size distribution as this can lead to a significant error in the attenuation estimate. This is illustrated in Figure 1 for which exponential drop size distributions of the form

$$N(D) = N_0 \exp\left(\frac{-3 \cdot 67 D}{D_0}\right) \quad (5)$$

were used to calculate directly, using the forward scattering amplitudes, both  $a_H$  and  $K_{DP}$  at the appropriate frequencies for a range of  $D_0$  and  $N_0$  values. The calculated  $K_{DP}$  was then used in equation (4) to find an estimate of the attenuation,  $a_{H(\text{estimate})}$ , and contours of the percentage error in the estimate, defined to be

$$\frac{a_{H(\text{estimate})} - a_H}{a_H} \times 100 \quad (6)$$

were drawn. Although the Marshall Palmer DSD has a value of  $N_0$  equal to  $8000 \text{ mm}^{-1} \text{ m}^{-3}$ , the contour of zero error is not a vertical line through this value because the power law of equation (4) is not exact but a least squares fit. It can be seen that an underestimate of the attenuation by 50% would not be an unreasonable occurrence.

It should be noted that in rain regions  $K_{DP}$  will always be positive and so this method of estimating attenuation is clearly limited to regions through which the measured  $\phi_{DP}$  is monotonically increasing. Since the measured  $\phi_{DP}$  includes

a contribution from backscatter phase, it can decrease in regions containing, for example, hail or ice particles.

### 3 RESULTS

To investigate the performance of equation (4) in practice, a second estimate of attenuation was obtained by comparison of the S- and C-band reflectivities. It was assumed that in regions of rain, if depolarization/attenuation effects were removed, the S-band circular reflectivity should not be significantly different from that of the C-band horizontal reflectivity. The S-band reflectivity was corrected for propagation effects using the method described in [5]. Where C-band reflectivity was observed to decrease relative to the S-band reflectivity over a number of bins, this decrease was attributed to the effects of attenuation. Regions of rain and, in particular, those where severe attenuation might be observed were found by identifying regions of rapidly increasing  $\phi_{DP}$ . Figure 2 shows an example of such a region from a storm on 21 July 1979 where  $\phi_{DP}$  increased from  $9^{\circ}$  to  $87^{\circ}$  over 10 bins or a two-way distance of 21 km. This gives an average  $K_{DP}$  through this region of  $3.7^{\circ}/\text{km}$ , which using equation (4) gives a total two-way attenuation through this region of 16 dB. A more exact bin-by-bin calculation gives the total attenuation to be 15.1 dB, which compares very well with the decrease seen in C-band reflectivity relative to that at S-band of 14.9 dB. Figure 2 also shows the S-band reflectivity after correction for propagation effects, the original C-band reflectivity and the C-band reflectivity corrected bin-by-bin using equation (4). Apart from two bins where the C-band reflectivity has been overcorrected by about 6 dBZ the two are in excellent agreement.

Such good agreement is not always seen as is illustrated in Figure 3 which uses data from storms occurring on 6 different days during the summer of 1979. Each data point shows the attenuation estimated using equation (4) bin-by-bin through a region of increasing  $\phi_{DP}$  plotted against that found by comparison of reflectivities. The solid line indicates where the two estimates are in agreement. The minimum number of bins over which the attenuation has accumulated is 5, while the maximum is 14. The maximum attenuation estimated by equation (4) was 18.9 dB which took place over 12 bins, while the maximum estimated from differences in reflectivity was 39.6 dB which took place over 11 bins.

In many cases equation (4) underestimates the attenuation with respect to that found from dif-

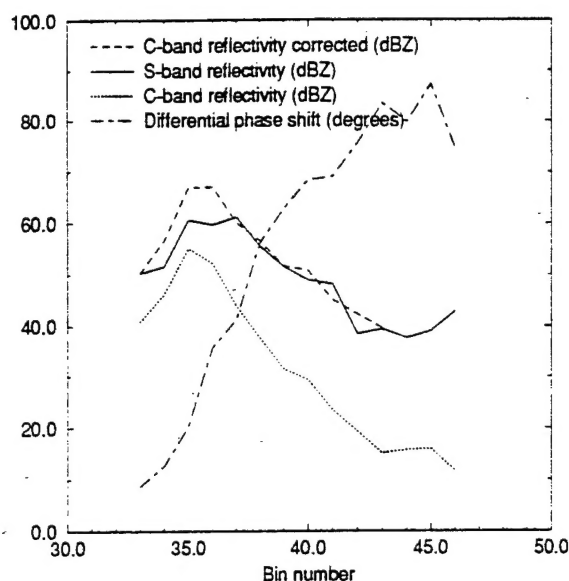


Figure 2: Data from storm of 21 July 1979

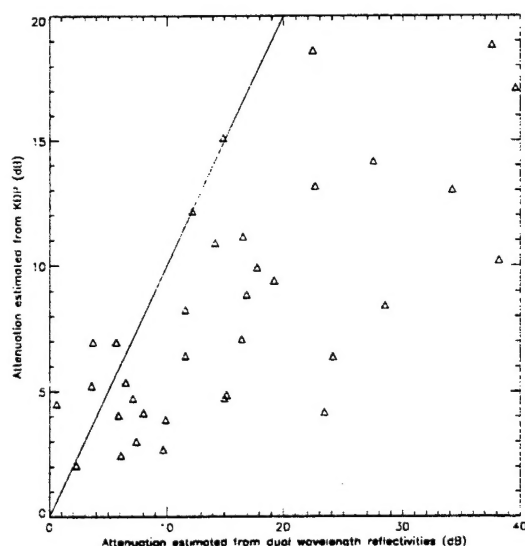


Figure 3: Comparison of two estimators of attenuation

ferences in the reflectivities. In view of Figure 1, however, this could be attributed to the fact that the DSDs are not, in reality, those of Marshall-Palmer.

Although in 1979 the polarization parameter data was not considered reliable enough to use in estimating drop size distributions, data from 1991 was available which could be used for this purpose. A DSD of the form given in equation (5), where  $N_0$  and  $D_0$  were free to take any values, was assumed. The parameter

$$Z_{DR} = 10 \log \left( \frac{Z_H}{Z_V} \right) \quad (7)$$

which can be derived from the circularly polarised data is independent of  $N_0$  and was used to determine the value of  $D_0$ .  $N_0$  was then determined using either  $Z$  or  $K_{DP}$ . It was found, however, that in many cases the estimate of  $N_0$  from  $Z$  did not agree with that from  $K_{DP}$ . This lack of agreement is illustrated in Figure 4 where  $Z_{DR}$  is plotted against  $Z_e/K_{DP}$ , where  $Z_e$ , the effective reflectivity factor, is in units of  $mm^6 m^{-3}$ . Both these quantities are independent of  $N_0$  and a theoretical curve calculated assuming an exponential DSD and a range of  $D_0$  from 0.5 to 5 mm has been plotted together with measured values from a storm on 26 July 1991. The measured values are all from regions of the storm through which  $\phi_{DP}$  was increasing. Also plotted are theoretical curves for gamma DSDs of the form

$$N(D) = D^\mu N_0 \exp \left( \frac{-(3.67 + \mu) D}{D_0} \right) \quad (8)$$

for  $\mu=2$  and  $\mu=5$ . It can be seen that the curve is insensitive to the form of DSD assumed and yet many of the points lie very far from it, with the values of  $Z_e/K_{DP}$  being much higher than expected. One possible reason for this is that the region may not be entirely composed of rain but may include hail or melting hail. Further investigation of this is required. The presence of hail could also explain the underestimate of attenuation by  $K_{DP}$  compared to that given by the difference in reflectivities. This is because, after removing propagation effects, the S- and C-band reflectivities would not necessarily be the same in a region containing hail.

Figure 5 shows a typical case where C-band reflectivity including attenuation effects has been calculated using an exponential DSD derived from  $(Z, Z_{DR})$  and from  $(K_{DP}, Z_{DR})$ . The measured C-band reflectivity is shown for comparison. It can be seen that using  $Z$  to derive  $N_0$  rather than  $K_{DP}$  leads to severely overestimating the attenuation observed, while using  $K_{DP}$  improves the situation but leads to an underestimate. Possibly, the presence of hail has led to the S-band

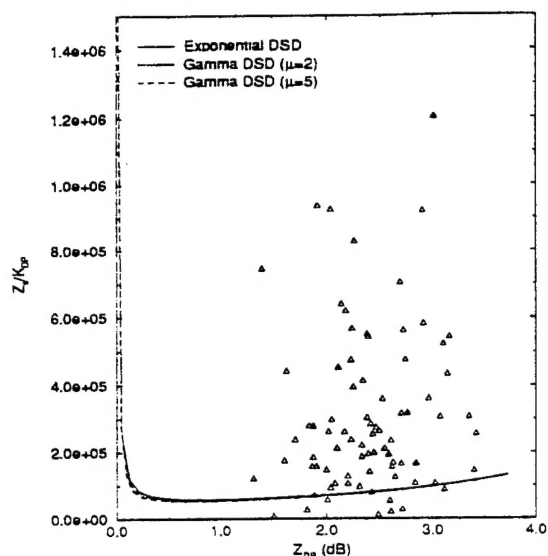


Figure 4: Comparison of theory and measured radar parameters for a storm on 26 July 1991

reflectivity being enhanced, while  $K_{DP}$  has been reduced.

#### 4 CONCLUSIONS

Significant attenuation has been observed at C-band by comparing the reflectivities measured by the two co-located radars. The empirical relationship derived by Tan [1] appears to predict attenuation well in some cases but often may be underestimating it. This may be due to the assumption of a drop size distribution of Marshall Palmer form, inherent in this equation, being an inappropriate choice. An attempt to verify this hypothesis was inconclusive as the three measured quantities of  $Z$ ,  $K_{DP}$  and  $Z_{DR}$  were not consistent with a model of either an exponential or gamma drop size distributions. This may be due to the presence of hail or melting hail, and further investigation is required.

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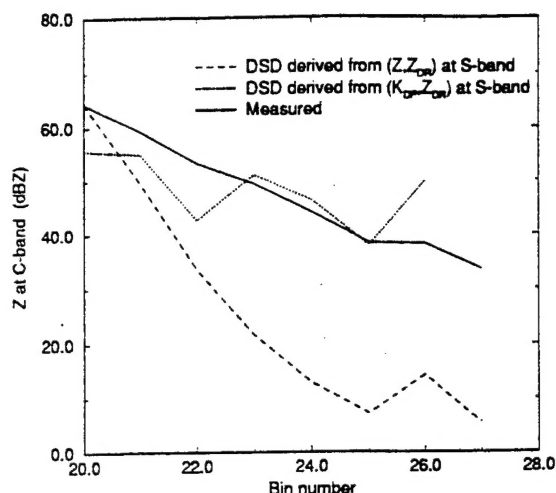


Figure 5: Measured C-band reflectivity compared with that predicted using drop size distributions derived from S-band measurements (from storm of 26 July 1991)

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